Joint Life Expectancy and the Retirement Distribution Period

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Executive Summary

- Past distribution research has focused primarily on determining sustainable withdrawal strategies based on fixed distribution periods, such as 30 years. While such research provides valuable information as to the likelihood of a portfolio failing over a predetermined period, it does not directly address the primary goal of retirees: to provide income for life.
- For most retirees, a retirement income strategy is likely to be deemed a failure only if it fails while either or both members of the retired couple is still living. Based on this logic, this paper will explore the implications of using joint life expectancy versus a fixed period when determining the appropriate initial sustainable real withdrawal rate for a distribution portfolio.
- Based on the research conducted for this paper, a sustainable withdrawal rate based on a joint expectancy period results in a 1–2 percent higher withdrawal rate for the same probability of failure than one based on a comparable fixed period.
- Using a target end date such as age 90, 95, or 100 is a simple method that can be used to determine an appropriate length for a distribution period for a retiring couple.

Previous distribution research has commonly used fixed distribution periods, such as 30 years, when determining an appropriate sustainable withdrawal rate. The fixed period is typically defined so that the likelihood of a retiree outliving it is small. After determining the length of the distribution period, an initial withdrawal rate is selected based on an acceptable probability of the portfolio accomplishing its goal (that is, not running out of money during the distribution period). While this two-step process is incredibly common, it ignores the possibility that the retiree (or a retiring couple) may die during the distribution period and the implications of this on the initial withdrawal rate.

For most retirees, a distribution strategy will be deemed to have failed only if it runs out of money during the lifetime of the retiree. If the retiree (or the retired couple) both die before a portfolio runs out of money, the portfolio has achieved its objective: to provide lifetime income. Common distribution metrics, though, often result in overly conservative withdrawal estimates when based on fixed periods. For example, as you will see later, while the probability of a portfolio with a 4 percent initial real withdrawal rate failing over a 30-year period is approximately 4 percent (regardless of the retired couple's age), the probability of a portfolio failing while either member of a couple, both age 65, are still living is slightly less than 1 percent.

This paper will explore the implications of using joint life expectancy versus a fixed period (30 years) when determining the appropriate initial sustainable real withdrawal rate. A simple methodology to determine an appropriate distribution period, based on target end date, will also be introduced.

Literature Review

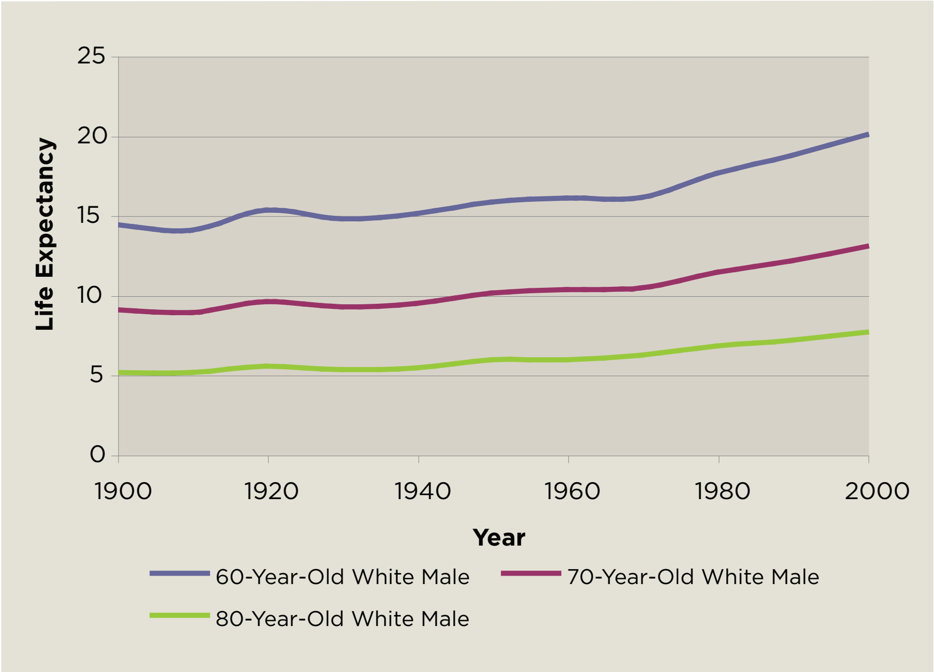
William Bengen provided some of the earliest research on sustainable withdrawal rates, noting that a "first-year withdrawal rate of 4 percent, followed by inflation-adjusted withdrawals in subsequent years, should be safe. In no past case has it caused a portfolio to be exhausted before

33 years" (Bengen 1994). He goes on to analyze the probability of different equity portfolios and concludes that the best starting allocation for retirees is an equity allocation between 50 percent and 75 percent, based on historical returns. Research by Tezel (2004), Cooley, Hubbard, and Walz (1998), Cassaday (2006), and Guyton and Klinger (2006) confirm the importance of 50 percent-plus equity allocations for distribution portfolios. But higher equity allocations also lead to riskier portfolios, which is why Terry (2003) concludes "that for a given level of wealth and specified level of risk, increasing the percentage held in equities actually decreases the initial percentage that can be withdrawn from a portfolio."

Bengen (1994) and Cassaday have also noted the importance of domestic small-cap equities for a distribution portfolio, and the long-term benefits of small-cap equities (as well as value) have also been well documented by French and Fama (1992, 1993, 1995, and 1996). The importance of international equities for distribution portfolios has differed among studies. Cooley, Hubbard, and Walz (2003) find moderate benefit from including international equities, while Kizer (2005) notes a greater benefit.

In an effort to increase the probability of achieving a particular withdrawal rate, a variety of decision rules and more advanced withdrawal strategies have been introduced by Pye (2000), Bengen (2001), Guyton (2004), Guyton and Klinger (2006), and Robinson (2007). Decision rules are relevant from a commonsense perspective because when faced with the possibility of financial ruin, it is likely that a retiree will decrease consumption to ensure continued survival of savings. But the ability to consistently follow a predetermined set of distribution rules over an extended period, such as 30 years, is questionable, especially for do-it-yourself clients. Dynamic and sophisticated decision rules are also not viable strategies for the generally unsophisticated investing public.

Figure 1: Life Expectancy for a White Male Based on Various Ages



Stout and Mitchell (2006) introduced the concept of life expectancy (mortality) into the sustainable withdrawal rate decision process. The authors found that introducing mortality into the planning horizon decreases the probability of portfolio failure by approximately 50 percent. They take the perspective, one which is followed in this paper, that "financial ruin should be defined as the probability of running out of money in the retirement life span, whether it be shorter or longer than a predetermined set of years." For example, Stout and Mitchell found that while a fixed 4.5 percent withdrawal rate has a 13.44 percent probability of ruin before 30 years, it only has a 7.16 percent probability of ruin before life expectancy (for someone 60 years old). This paper is an extension of their work.

Changing Life Expectancies

Life expectancy is defined as the number of additional years an individual is expected to live beyond a given age. The concept of life expectancy is an extremely important assumption in a financial plan since the length of the distribution period (that is, the number of years to provide income for retirement) is going to be based on the life expectancy of the retiree, or the joint life expectancies of the retiree and spouse. Life expectancies have increased considerably over the last 100 years, as shown in Figure 1, which includes the historical life expectancies for white males ages 60, 70, and 80.

Since 1950, for a 60-year-old white male the average life expectancy has increased 1 month a year, 0.7 months a year for a 70-year-old white male, and 0.4 months a year for an 80-year-old white male. (While information on newborns is not included in Figure 1, primarily because saving for retirement is not a particular concern for newborns, the average life expectancy for newborns has been increasing by approximately two months a year since 1950.)

Life expectancies are always changing and are different for various socioeconomic and gender groups. Some common long-term life expectancy trends are that females tend to outlive males, and whites tend to outlive other ethnic groups. Studies have also shown that individuals with

Table 1: Probability of One or Both Spouses Living Additional Number of Years Past Their Current Ages (Based on 2004 Periodic Life Table)

Current Age	Number of Years Past Current Age									
	5	10	15	20	25	30	35	40	45	50
50	99.9%	99.6%	98.8%	96.6%	91.6%	81.5%	63.6%	38.0%	14.3%	2.9%
55	99.9%	99.2%	97.3%	92.6%	82.7%	64.7%	38.8%	14.6%	3.0%	0.3%
60	99.7%	98.2%	94.0%	84.4%	66.4%	40.0%	15.1%	3.0%	0.3%	0.0%
65	99.3%	95.9%	87.1%	69.2%	41.9%	15.9%	3.2%	0.3%	0.0%	0.0%
70	98.3%	90.9%	73.4%	45.1%	17.2%	3.5%	0.3%	0.0%	0.0%	n/a
75	96.1%	80.2%	50.5%	19.5%	3.9%	0.4%	0.0%	0.0%	n/a	n/a
80	90.5%	60.1%	23.8%	4.8%	0.5%	0.0%	0.0%	n/a	n/a	n/a
85	77.9%	33.0%	6.8%	0.7%	0.0%	0.0%	n/a	n/a	n/a	n/a
90	56.4%	12.3%	1.2%	0.0%	0.0%	n/a	n/a	n/a	n/a	n/a
95	33.3%	3.4%	0.1%	0.0%	n/a	n/a	n/a	n/a	n/a	n/a

Age 90 as Target | Age 95 as Target | Age 100 as Target

Source: Social Security Administration

higher lifetime earnings and more education tend to have longer life expectancies than those with lower earnings and less education. This is an important consideration since most individuals seeking the professional help of a financial planner tend to have higher-than-average levels of earnings and education, and therefore higher-than-average life expectancies.

Defining Success

To determine the appropriate sustainable real withdrawal from a distribution portfolio, two distinct but interrelated probabilities must be determined. First, we must decide on the acceptable probability of outliving the distribution period. The distribution period is typically set so the likelihood of outliving it is small, whether it is based on a single person or joint couple. For example, if a couple (male and female), both age 65, wanted no more than a 10 percent probability that either one would outlive the distribution period, the distribution period would need to be 32 years (to age 97, based on the 2004 Period Life Table obtained from the Social Security Administration's Web site).

After the distribution period has been determined, the acceptable probability of failure during the distribution period must be determined. Again, if the couple wanted no more than a 10 percent probability that the portfolio would fail during the 32-year distribution period, the sustainable real withdrawal rate would need to be approximately 4 percent, based on a 60/40 portfolio (60 percent equity, 40 percent fixed income and cash). This information can be found in Table 2 on page 59. But the likelihood that a 60/40 portfolio with a 4 percent real withdrawal rate would fail while either member of a 65-year-old couple is still living is not 10 percent—it's actually slightly less than 1 percent (see Table 3 on page 59).

In other words, there is only a 0.7 percent probability that a portfolio with a 4 percent initial real sustainable withdrawal rate would fail while either member of the couple is still living, versus the target 10 percent used to determine the withdrawal amount based on the fixed period. This suggests that the traditional method of determining the sustainable real withdrawal rate may lead to overly conservative withdrawal estimates for retirees. If that 65-year-old couple wanted a sustainable withdrawal rate with a 10 percent probability of failure based on their life expectancy (versus a fixed period), the withdrawal rate could be increased from 4 percent to 5.5 percent (this is estimated from Table 3).

The retirement distribution period for married couples is typically based on the likelihood of either member of the couple outliving the period (not a single member, which was the approach taken by Stout and Mitchell). Using the joint life expectancy of a couple versus the single life expectancy of an individual can increase the length of distribution period by varying degrees, depending on the respective ages of the spouses.

For example, for a joint couple retiring at age 65, the probability of a male living to age 95 (30 years) is 5.3 percent, while the probability of a 65-year-old female living to age 95 (again 30 years) is 11.36 percent. The probability of one (or both) of the spouses living to age 95, however, is 15.9 percent (based on the 2004 Period Life Table obtained from the Social Security Administration's Web site). To help the reader better understand the likelihood of either member of a joint couple outliving a certain period, Table 1 lists the probability that at least one spouse of a male and female couple, both the same age, will live an additional number of years past their current age.

Table 1 demonstrates that a "one size fits all" approach to the distribution period will not result in a reasonable estimate for each retiree. For example, while a 30-year distribution period may seem reasonable for a 65-year-old couple (who have a 15.9 percent probability of outliving the 30-year period), it is materially less so for a couple both age 55 (with a 64.7 percent probability of outliving the period) or a couple both age 75 (with a 0.4 percent probability of outliving the period). Clearly a 30-year distribution period for a 55-year-old couple is too short, while a 30-year distribution period for a 75-year-old couple is too long. This is why the unique ages and health attributes of each retired couple need to be considered and why there is more than one retirement period worth considering.

Table 1 also demonstrates the advantage of using target dates when attempting to determine the length of the distribution period for a retiring couple. A financial planner could use age 90, 95, or 100 as the target ending age for a retiring couple, where the length of the distribution period could be determined by subtracting the couple's joint age from the target date. For example, for a couple both 65 years old, the distribution period would be 30 years based on a target end date of 95 (95 – 65 = 30). For a couple who are different ages, one of two approaches can be used. The first is to take the average ages of the spouses to determine the joint age, while the second and more conservative approach is to base the joint age on the youngest member of the couple, which is typically going to be the female. For the most common retirement periods (ages 60–70), the probability of either or both spouses living past age 90 is about 42 percent, the probability of either or both spouses living past age 95 is about 16 percent, and the probability of either or both spouses living past age 100 is about 4 percent.

The higher the target age (age 100 versus age 95), the longer the distribution period and therefore the more conservative the period estimate. The relative health of a couple could be used to select the appropriate target age. For example, a target age of 90 could be used for a couple with below-average health, a target age of 95 could be used for a couple with average health, and a target age of 100 could be used for a couple with above-average health. Some additional considerations, though, are that retirement spending needs to decrease at older ages (85-plus) and retirees with significant assets tend to have better access to health care, which increases life expectancy.

Analysis

An analysis was conducted to determine the implications of joint life expectancy on a distribution portfolio. It is assumed, for the analysis, that the clients hold a 60/40 portfolio for the entire distribution period. While the authors realize that retirees have varying risk preferences, 60/40 is a common allocation for defined-benefit plans, and its aggregate benefits for a distribution portfolio have been noted in previous research (Blanchett 2007). The overall allocation of the 60/40 portfolio was 20 percent cash, 20 percent intermediate-term bond, 40 percent domestic large blend equity, and 20 percent international equity.[1]

To assume a constant distribution rate (thereby removing the need to consider inflation), the monthly returns were adjusted into real terms by subtracting the monthly inflation rate, which was defined as the increase in the Consumer Price Index for All Urban Consumers (CPI-U). CPI-U was used because it is the most common definition of inflation. All data were obtained from the Bureau of Labor Statistics.

The actual returns used for testing purposes were created through bootstrapping. (Bootstrapping is a type of simulation analysis where the in-sample test period returns are randomly recombined to create annual returns for testing purposes.) For the analysis, monthly return information was obtained from 1927 to 2007 (81 calendar years) and randomly recombined to create hypothetical real annual rates of return for the analysis. For example, the monthly real returns for each of the four categories for the same month (say, June 1961) would be recombined with monthly real returns from 11 other months (for example, March 1930, January 1995, May 1979, and so on) to create each hypothetical annual real return. A benefit of the bootstrapping process is that no assumptions need to be made about the distribution of hypothetical returns (that is, leptokurtic and positively skewed).

Distributions from the portfolio are assumed be taken from the portfolio at the beginning of each year (once a year). Each test scenario was subjected to a 10,000-run Monte Carlo simulation. The Monte Carlo simulator (that is, bootstrap simulator) used for the analysis was built in Microsoft Excel from scratch. For each run, the portfolio return and the male and female life expectancies were separately randomized. A portfolio was considered successful if it did not run out of money either during a fixed distribution period (say, 30 years) or if it survived the distributions as long as at least one spouse (of the couple) was still alive (depending on test).

The portfolios were assumed to be held in tax-deferred accounts and any tax implications of the withdrawals were ignored. Based on the bootstrapping methodology, it was implicitly assumed that the portfolios were rebalanced monthly back to their target allocations. Any potential costs associated with the rebalancing were ignored. Tax considerations were also ignored for the analysis.

The 2004 Periodic Life Table, obtained from the Social Security Administration's Web site, is used for life expectancy calculations. For the analysis, it is assumed that a male and female couple retire at the same time and are both the same age. For each year of each run, the probability of either spouse dying is considered for each of the 10,000 runs (separately on an annual basis). If both spouses die and the portfolio still has a positive account value, the portfolio is considered successful. Each Monte Carlo run is considered failing only if the portfolio value drops below $0 while either or both spouses are still living.

Results

Because portfolio failure is typically defined based on a fixed distribution period, the fixed period results will be reviewed first. Table 2 includes information on the likelihood of failure for a 60/40 portfolio based on a variety of distribution periods and initial real withdrawal rates. The distribution periods (the x-axis of Table 2) correspond to the joint ages used in Table 3 based on age 100 as the target end date. This is why the distribution periods are listed in descending order versus ascending order.

Not surprisingly, the distribution period and the withdrawal rate have a considerable impact on the probability of failure for a distribution portfolio. For example, a 30-year distribution period and a 4 percent real distribution rate has a 4.1 percent probability of failure, while a real withdrawal rate of 6 percent for a 30-year distribution period has a 39.0 percent probability of failure.

Table 2: Probability of 60/40 Portfolio Failure for Various Real Withdrawal Rates and Distribution Periods

Initial Real Withdrawal Rate	Distribution Period (Years)								
	50	45	40	35	30	25	20	15	10
2.00%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
3.00%	2.9%	2.2%	1.4%	0.8%	0.3%	0.1%	0.0%	0.0%	0.0%
4.00%	16.9%	14.2%	11.0%	7.5%	4.1%	1.4%	0.2%	0.0%	0.0%
5.00%	39.1%	35.4%	30.4%	24.2%	16.4%	8.2%	2.1%	0.1%	0.0%
6.00%	63.8%	60.5%	55.8%	48.8%	39.0%	25.6%	10.4%	1.0%	0.0%
7.00%	83.8%	81.8%	78.7%	73.8%	65.8%	52.5%	30.8%	6.7%	0.0%
8.00%	93.8%	92.8%	91.3%	88.7%	84.1%	74.9%	55.2%	20.4%	0.3%
9.00%	98.3%	97.9%	97.4%	96.4%	94.4%	90.0%	77.8%	43.8%	2.0%
10.00%	99.4%	99.2%	99.0%	98.7%	97.9%	96.1%	90.3%	66.7%	7.9%
11.00%	99.8%	99.8%	99.7%	99.6%	99.4%	98.8%	96.5%	84.0%	21.9%
12.00%	99.9%	99.9%	99.9%	99.9%	99.8%	99.5%	98.6%	92.3%	40.1%

Table 3: Probability of 60/40 Portfolio Failure While Either or Both Members of a Couple Are Still Living

Initial Real Withdrawal Rate	Joint Age of Couple (Years)								
	50	55	60	65	70	75	80	85	90
2.00%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
3.00%	0.4%	0.2%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%
4.00%	4.1%	2.9%	1.7%	0.7%	0.3%	0.1%	0.0%	0.0%	0.0%
5.00%	17.4%	12.8%	8.3%	4.8%	2.3%	0.7%	0.1%	0.0%	0.0%
6.00%	39.4%	32.8%	24.7%	15.8%	8.6%	3.5%	1.0%	0.1%	0.0%
7.00%	62.5%	55.2%	45.2%	33.3%	20.6%	9.9%	3.2%	1.0%	0.2%
8.00%	79.8%	74.0%	64.6%	52.0%	36.4%	20.8%	8.9%	2.6%	0.6%
9.00%	90.5%	86.1%	78.6%	68.7%	52.9%	34.9%	17.1%	5.7%	1.3%
10.00%	95.6%	92.5%	87.9%	80.7%	66.5%	47.0%	26.5%	10.3%	3.0%
11.00%	98.0%	96.1%	93.3%	87.6%	76.7%	59.3%	36.2%	16.2%	5.3%
12.00%	98.8%	97.9%	95.8%	91.5%	82.9%	68.2%	46.3%	23.3%	8.2%

The decision to base a sustainable real withdrawal rate entirely on a fixed distribution period, though, ignores the likelihood of one or both members of a couple dying during the distribution period. The goal of most retirees is to create an income stream that will last their lifetimes (ignoring any type of bequest motives, which is commonly a secondary consideration to generating income for life). If both spouses die during their first year of retirement and the portfolio runs out of money one year into retirement, the portfolio may have "failed" during the presumed fixed distribution period, but it still achieved its goal (that is, generating income for the couple's life). Table 3 includes the same basic information as Table 2; however, a portfolio is considered to be failing for the results in Table 3 only if either or both spouses are still alive when the portfolio runs out of money. Failure in Table 2, on the other hand, is based on failing during a fixed distribution period.

When joint life expectancy was used as the distribution period instead of a fixed period (where age 100 is the target end date), it always resulted in a lower probability of failure (that is, higher probability of success) for each of the selected test scenarios (unless both probability of failure for each test was 0 percent).

To put Table 3 in perspective, let's revisit the original example of the 65-year-old couple. Let's say you wanted no more than a 1 percent probability of failure, defined as failing over a fixed 30-year period. This would result in a 3 percent initial real withdrawal rate (Table 2). But when the probability of failure is defined using joint life expectancy, the withdrawal rate increases to 4 percent (Table 3). Note that the target probability of failure is the same for both scenarios (1 percent), but using the fixed distribution actually overstates the probability of the portfolio failing during the couple's lifetime.

While the differences between using joint life expectancy and a fixed distribution period varied among the different scenarios, the real withdrawal rates tended to be between 1 percent and 2 percent higher when based on joint life expectancy versus a fixed distribution period, especially for the more conservative scenarios. Note that the differences tended to be greater for higher real withdrawal rates and over shorter distribution periods than for lower real withdrawal rates and over longer distribution priods. In other words, if it is determined that a couple's sustainable real withdrawal rate would be 4 percent using a fixed period, it would likely correspond to a withdrawal rate between 5 percent and 6 percent based on joint life expectancy.

Viewed differently, using a shorter fixed distribution period can generate a result similar to using joint life expectancies. If the target distribution end date is reduced to age 89, the probability of failure for the two strategies is roughly the same. In other words, using a more aggressive assumption for the target end fixed period (age 89) versus a more conservative estimate (such as age 95 or age 100) replicates the portfolio rates when joint life expectancy is used. Using the age 89 approach may be necessary for those financial planners without Monte Carlo generation tools that can incorporate joint life expectanices in distribution studies.

Conclusion

While using a fixed period is mathematically simpler than using life expectancy data when conducting a distribution analysis for a retiree, it ignores the primary goal of most retirees: to not outlive their savings. The purpose of this paper has been to explore the differences between using the joint life expecatancy of a couple and a fixed period when determining an appropriate sustainable real withdrawal rate from a distribution portfolio. Using joint life expectancy tended to result in a sustainable real withdrawal rate that was between 1 percent and 2 percent greater than the sustainable real withdrawal rate available from a portfolio based on a fixed distribution period for the same probability of failure. Therefore, the sustaianble real withdrawal rate available based on a fixed time period is likely an overly conservative estimate for most retirees.

Endnote

1. Data definitions:
   - Intermediate-term bond: defined as the return on the Moody's Seasoned Aaa Corporate Bond Yield, assuming a ten-year duration. Data obtained from the St. Louis Federal Reserve: research.stlouisfed.org/fred2/.
   - Cash: defined as the yield on the three-month Treasury bill. Secondary Market Rate, data obtained from Tradetools.com (1927–1933) and the St. Louis Federal Reserve (1934–2006): research.stlouisfed.org/fred2/.
   - Domestic large blend equity: defined as the return on the "Big Neutral" portfolio based on the 2 × 3 portfolio return information publicly available on Kenneth French's Web site: mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.
   - International equities: defined as the return on the Global Financial Data World ex-USA Return Index, data obtained from Global Financial Data.

Because pure historical data are used for this analysis, as is common among distribution research, the authors would caution the reader that if future returns are lower than historical returns, the actual result of a distribution portfolio may be materially different from what this research suggests.

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